Random Holomorphic Functions

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ABSTRACT

A holomorphic function in the unit disk $\{|z| < 1\}$ is a complex valued function $w = f(z)$ having a convergent power series

$$w = f(z) = \sum_{n=0}^{\infty} f_n z^n.$$

To model randomness we choose a probability measure on the set (actually an algebra) of all such functions. There is not a preferred choice, but symmetry is always desirable. We will discuss two questions, prefaced with some motivation.

First, can we classify all of the measures which have a natural (conformal) symmetry, i.e. can we find all invariant measures (a question which arises in many contexts)? The answer appears to hinge on some examples which we will consider. Secondly, some measures (e.g. Gaussian measures) are more interesting than others. For an interesting notion of randomness, can we say something about the zero set of a random holomorphic function, which is a random configuration of points in the disk (like stars in the universe)?